A Bound on Log Likelihood from Lyapunov Exponents

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In 1977 Pesin[1] proposed a kind of relation between Lyapunov exponents of differentiable dynamical systems and their entropy rates. Subsequent work by many investigators, including Young[2], proved that similar relations hold for many classes of systems. The *Pesin relation* suggests a bound on the log likelihood for any probabilistic model of time series generated by a chaotic system. In this poster, I summarize numerical experiments[3] in which I calculated a sequence of log likelihoods for time series from the Lorenz system[4]. Each element of the sequence represents the performance of an HMM with more hidden states than its predecessors. I found that it takes a remarkably large number of hidden states to approach the bound.

References

[1] Ya. B. Pesin. Characteristic lyapunov exponents and smooth ergodic theory. *Russ. Math. Surv.*, 32(4):55–112, 1977.

[2] L. Young. Ergodic theory of differentiable dynamical systems. In B. Branner and P. Hjorth, editors, *Real and Complex Dynamical Systems*. Kluwer, 1995. http://citeseer.ist.psu.edu/young95ergodic.html.

[3] Andrew M. Fraser. *Hidden Markov Models and Dynamical Systems*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 2008.

[4] E. Lorenz. Deterministic nonperiodic flow. *J. Atmos. Sci.*, 20, 1963.

Topic: graphical models
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